Consider a dielectric slab waveguide made of a nonmagnetic isotropic material. The slab has a width $2b$ along the x direction. The slab is infinitely extended in the y and z directions. The refractive index is given by:

$$n(x)=\begin{cases} n_1, & |x|\le a \\ n_2, & |x|>a \end{cases}$$

where $a<b$ and $n_2<n_1$. Now solve Problems 1,2, and 3.

1) Starting from Maxwell's equations, derive the total electromagnetic field equations in the region $|x|\le a$.

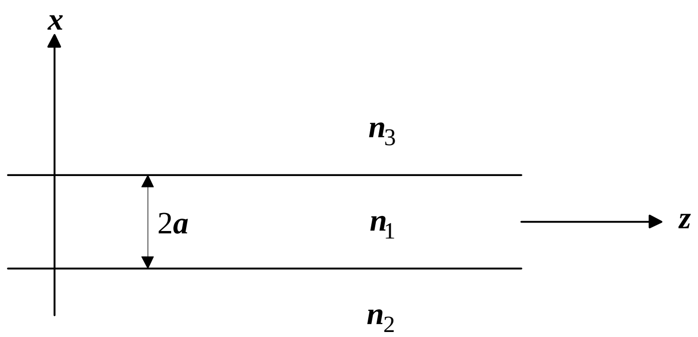
2) Let $a=25\ \mu m,\ n_1=1.5,\ n_2=1.49$. Determine the cut-off wavelengths of all modes up to order 5.

3) If the normalized frequency is equal to 3.0 radians, specify and sketch all supported TE modes.

4) An optical fiber has a core and cladding refractive indices $n_1=1.5, n_2=1.4$. Let the fiber length be 1 km. The fiber has an attenuation rate of 2 db/km. Give only the final answer of each of the following questions in the specified space.

4.a.	What is critical angle?	
4.b.	What is acceptance angle?	
4.c.	What fraction of input power is lost in 500 m?	

5) The figure below depicts an asymmetric planar dielectric slab waveguide. Derive the TE mode equations in this guide.



$n_1>n_2>n_3$

6) Consider an optical fiber of core diameter $2a$ and cladding diameter $2b$. Let the refractive index profile in the fiber be given by:

$$n(r)=\begin{cases} n_1, & 0\le r<r_1 \\ n_1-\delta, & r_1\le r<2r_1=a \\ n_1-2\delta, & a\le r\le b \end{cases}$$

6.a. What is the acceptance angle?

6.b. What is the critical angle?

6.c. What is the ratio of the longest ray path length to the shortest ray path length? (In this question ***only***, let $\delta = 0.1n_1$)

6.d. What happens to the critical angle as δ is increased?

6.e. Knowing that $\delta = \xi n_1$, what is the range of n_1 values that can allow light to propagate along the fiber?

6.f. What happens to the delay dispersion as δ is increased?

7) Consider a graded index optical fiber with a cladding refractive index n_2 Let the core refractive index profile be given by:

$$n(r) = n_1 \left(1 - A\left(\frac{r}{a}\right)^{\alpha}\right)^{1/2}.$$

7.a. Determine the constant A in terms of n_1, n_2 and α.

7.b. Assuming $n_1 = 1.5$, $n_2 = 1.46$ and $\alpha = 1$, sketch the refractive index profile.

7.c. Repeat the last part when $\alpha = 2$ and $\alpha = 3$.

8) A semiconductor laser has a $500\ \mu\text{m}$ length. The active region material has 1.43 eV band gap energy and 13.2 relative permittivity. The gain in the active region is given by:

$$g(\lambda) = g(\lambda_0)\, e^{-(\lambda-\lambda_0)^2/2\sigma^2}$$

where λ_0 is the wavelength of peak gain, and σ is the gain spectrum width. Assume the peak gain $g(\lambda_0) = 5000\,/\,\text{m}$ and the threshold gain $\alpha_t = 4000\,/\,\text{m}$. Let $\sigma = 28\ \text{nm}$.

8.a. Sketch the gain spectrum

8.b. Determine the central emission wavelength

8.c. Determine the minimum and maximum emission wavelengths

8.d. Determine the lasing frequency separation

8.e. Determine the lasing wavelength separation

8.f. Determine the number of supported wavelengths

8.g. Sketch the lasing spectrum as function of frequency

8.h. Sketch the lasing spectrum as function of wavelength

Electron Charge: $q = 1.602 \times 10^{-19}\,\text{C}$ Planck's Constant: $h = 6.626 \times 10^{-34}\,\text{J.s}$