1) An analog source produces a baseband voltage signal $x(t)$ with bandwidth equal to 10 kHz. Assume that sample functions of $x(t)$ follow a probability density given by:

$$f_X(x) = \begin{cases} \frac{1}{2}e^{-x/2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

The source output is quantized according to the rule:

$$y_i = \begin{cases} i - 0.5, & i-1 \le x < i, \text{ for } 1 \le i \le 7 \\ 8.0, & x \ge 7.0, i = 8 \end{cases}$$

Each quantized level y_i is denoted as a discrete source symbol g_i for $i = 1, 2, \ldots, 8$.

a. Determine the symbol probabilities.

b. What is the minimum required bit rate of a fixed-length encoder for the quantizer output?

2) A 4-symbol baseband digital communication system uses the waveforms below to represent its symbols, each of duration T.

$$s_1(t) = \begin{cases} 1, & 0 \le t \le T/2 \\ 0, & \text{elsewhere} \end{cases} \qquad s_2(t) = \begin{cases} 1, & T/2 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$$

$$s_3(t) = \begin{cases} 1, & 0 \le t \le T/2 \\ -1, & T/2 \le t \le T \end{cases} \qquad s_4(t) = \begin{cases} -1, & 0 \le t \le T/2 \\ 1, & T/2 \le t \le T \end{cases}$$

Let $\varphi_1(t) = s_1(t) \big/ \sqrt{E_{s_1}}$ and $\varphi_2(t) = s_2(t) \big/ \sqrt{E_{s_2}}$.

a. Determine the signal space vectors.

b. Sketch the signal space representation of the four signals.

c. Determine the signal energies and the average symbol energy.

d. Determine the distances between all signal vector pairs.

e. Determine the cross correlations between all signal vector pairs.

3) Determine the average symbol and bit energies of 16-ASK, if the distance between adjacent signal space points is equal to $2d$.

4) If an analog signal with bandwidth 5000 Hz is sampled at the Nyquist rate, what

would be the bit rate a 4 bit/sample DPCM encoder? What would be the bit rate of a delta modulator used with the same signal?

5) A set of ***M*** pulse duration modulation (PDM) signals are given by:

$$s_i(t) = \begin{cases} A, & 0 \le t < iT/M \\ 0, & \text{otherwise} \end{cases}, \quad i = 1, 2, \ldots, M$$

a. Sketch the signals and determine their energies.

b. Assuming M=4 and equal symbol probabilities determine the average symbol energy.

c. Determine the cross-correlation coefficient between any two signals $s_i(t)$ and $s_j(t)$.

d. What is the dimensionality of the signal space?

e. Sketch the signal space for ***M***=2 and for ***M***=3.

6) A set of *M*=4 signals ($i = 1, 2, 3, 4$) are given by $s_i(t) = A\sqrt{2/T}\cos(\pi i t/2T)$, where *A* and *T* are constants. Assume equal symbol probabilities.

a. Sketch the signals and determine their energies.

b. Determine the average symbol and bit energies.

c. Suggest a set of orthonormal basis functions to represent these signals.

d. Sketch the signal space.

7) Sketch the optimum receiver and write down the optimum decision rules for orthogonal 3-FSK in AWGN.

8) Consider a binary antipodal system. Sketch a matched filter receiver with the smallest possible number of matched filters. Determine the optimum decision rule.

9) Consider a baseband signal $s(t)$ of duration T. Let $h(t)$ be a filter matched to $s(t)$. Let $y(t)$ be the matched filter output when the filter input is $s(t)$. Determine the maximum filter output in terms of the input signal energy.

10) Consider the optimum receiver of a binary antipodal digital communication system operating in AWGN. The transmitter sends either $s_1(t)$ or $s_2(t)$ every T. Let the only basis function be defined by:

$$\varphi_1(t) = \begin{cases} 1, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$

The optimum receiver includes only one matched filter. Sketch the matched filter when $s_1(t)$ is transmitted. Sketch the matched filter when $s_2(t)$ is transmitted.